surface; U_0, P_0, initial velocity and pressure; U_K, final velocity; and θ_0, time of gate activation.

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STEADY-STATE TWO-DIMENSIONAL WAVES ON VERTICAL LIQUID FALLING FILMS AND THEIR STABILITY

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UDC 532.51

An investigation is reported of the stability of nonlinear conditions with respect to infinitely small perturbations; there is good agreement with experiment.

Their large interfacial contact areas and small thermal resistances make liquid films an effective means of carrying out interphase heat and mass transfer processes. It is well known that as a result of the instability of flows with flat free surfaces the nature of the motion of liquid films flowing down vertical walls is wavy even at small Reynolds numbers. The urgency to study these conditions arises as a result of the fact, in particular, that the presence of the waves has a considerable effect on the process of interfacial transfer through the free surface. Thus, in the desorption of slightly soluble gases the mass transfer coefficients may be increased by 100% or more as a result of the waves [1].

A special but important form of wavy flow consists of planar, steady-state periodic travelling waves. Their theoretical consideration is quite complicated, since it is necessary to solve a highly nonlinear boundary-value problem with a free boundary whose position is not known in advance. With the assumptions that the profiles of the longitudinal velocities are similar for any cross section x and any moment of time t:

$$u = 1{,}5\,\frac{q(x,t)}{h(x,t)}\left(2\,\frac{y}{h(x,t)} - \frac{y^2}{h^2(x,t)}\right)$$

and that the wavelengths are large, a system of equations has been derived in [2] for describing the behavior of perturbations on a film at moderate Reynolds numbers:

$$\frac{\partial q}{\partial t} + 1{,}2\,\frac{\partial}{\partial x}\left(\frac{q^2}{h}\right) = -\frac{3\nu q}{h^2} + gh + \frac{\sigma h}{\rho}\frac{\partial^3 h}{\partial x^3},\quad \frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0, \tag{1}$$

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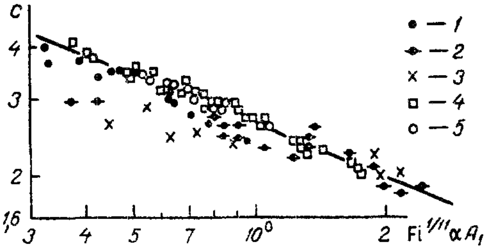

Fig. 1. Phase velocity c as a function of the amplitude and wave number: 1-4) experiments; 5) calculations; 1) data of [8]; 2) data of [10]; 3, 4) data of [9]; 5) results of [6, 7].

where q is the instantaneous liquid flow rate in the cross section x; h is the instantaneous film thickness.

A well-known result is obtained from the linearized system (1): infinitely small perturbations of the form $\exp[i\alpha(x - ct)]$ are unstable for values $\alpha < \alpha_N$.

The periodic nonlinear steady-state solution $h = h(x - ct)$, $q = q(x - ct)$ of system (1) has been investigated in [2-7]. In [2], the solution is represented in the form of a Fourier series, and by means of the first two harmonics it was shown that a family of wavy solutions branches off from the plane-parallel solution at $\alpha = \alpha_N$. In the limit, when $\alpha \to 0$, it is transformed into negative solitons [3], for which $\int_{-\infty}^{\infty} (h - h_\infty)dx < 0$, where h_∞ is the thickness of the unperturbed film. It is shown in [4] that from this family of curves (termed the first family in what follows) a second family of wavy solutions branches off in turn. From the point of view of comparing the calculations with experiments, the second family of curves which branches from the first family in the neighborhood of the point $\alpha = \alpha_N/2$ is of considerable interest, as well as the first family of curves. The waves of this type have in the limit a positive soliton solution $\int_{-\infty}^{\infty}(h - h_\infty)dx > 0$. For waves of the first type, $|\Delta h_{max}| < |\Delta h_{min}|$, while for waves of the second type, $|\Delta h_{max}| > |\Delta h_{min}|$, where Δh is measured from the mean level.

In the experiments the waves of the first type are usually close in shape to sinusoidal waves. Highly nonlinear, nonsinusoidal waves with amplitudes comparable to the mean film thickness belong to the second type as a rule.

A comparison is made in [5-7] of the wavy solutions of the system (1) with the experimental data, and good quantitative agreement is found. Thus, for example, Figure 1 (taken from [8]) demonstrates the agreement between our calculations for waves of the second type and the experiments of various authors. The equation of the general curve has the form $c = 2.6(Fi^{1/11}\alpha A_1)^{-0.4}$ [8].

In the experiments with natural flow the incipient waves are two-dimensional initially. They grow rapidly in amplitude, arriving at the steady-state at some distance from the point of entry of the liquid. Subsequently they break up and become three-dimensional. The use of artificially imposed perturbations considerably widens the range of wave numbers for which regular, two-dimensional waves are possible. This widening occurs at the lower boundary: the conditions are such that waves of longer wavelengths appear [9, 11]. The experiments which have been carried out show that the natural waves are a special case of the excited waves. When the frequencies coincide, they are identical to each other. The excited waves also become three-dimensional, though this occurs considerably further from the inlet than in the case of natural flow [9, 11].

The first problem of the stability of the wavy regime in a falling film was considered in [12]. The waves of the first type derived in [2] were used for this purpose. As a result of this study it was found that this regime of flow was unstable to two-dimensional perturbations for all values of the Reynolds number which were considered ($Re \gtrsim 30$).

A study was made in [13] of the stability of solutions of the equations describing the wavy regime at small values of the Reynolds number, $Re \lesssim 1$. Here a narrow zone of stability was found for conditions which branch off from those of planar, wave-free flow at α_N. In order to explain the results of the experiments and to make comparisons among the data of

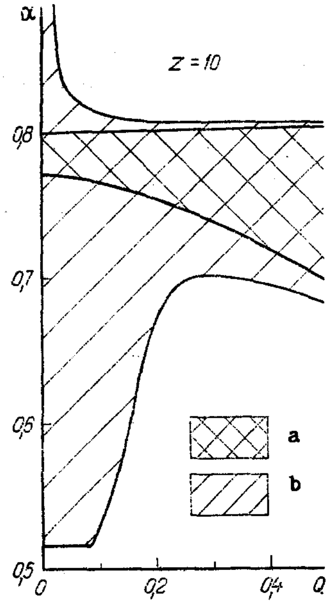

Fig. 2

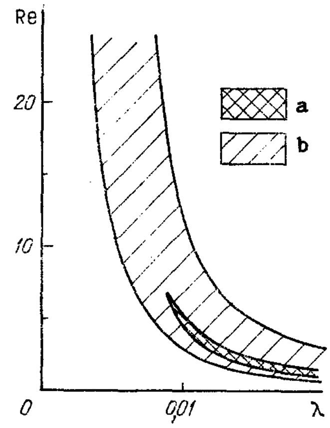

Fig. 3

Fig. 2. Zone of stability (a) and zone of slowly growing perturbations (b).

Fig. 3. Zones of stability to all two-dimensional perturbations (a) and with respect to perturbations with Q = 0 (b). λ is given in m.

[12, 13], let us investigate the stability of the solution obtained for the system (1) using the procedures proposed in these papers.

First, by using the mean values ⟨h⟩ and ⟨q⟩ over the wavelength as reference quantities for making the variables dimensionless, using the substitutions

$$\bar{h} = h/\langle h \rangle,\ q = q/\langle q \rangle,$$

$$\bar{x} = \left(\frac{\mathrm{We}}{3}\right)^{-1/2} x/\langle h \rangle,\ \bar{t} = \left(\frac{\mathrm{We}}{3}\right)^{-1/2} \langle h \rangle^2 t/\langle q \rangle,$$

$$\bar{\xi} = \bar{x} - \bar{c}\,\bar{t},$$

and omitting the signs when the quantities are made dimensionless, the system of equations (1) can be transformed to

$$\frac{\partial q}{\partial t} - c\frac{\partial q}{\partial \xi} + 1{,}2\frac{\partial}{\partial \xi}\frac{q^2}{h} = -z\frac{q}{h^2} + Fh + 3h\frac{\partial^3 h}{\partial \xi^3},$$
$$\frac{\partial h}{\partial t} - c\frac{\partial h}{\partial \xi} + \frac{\partial q}{\partial \xi} = 0, \qquad (2)$$

where $z = (3\mathrm{We}/\mathrm{Re}^2)^{1/2}$, $F = (\mathrm{We}/3\mathrm{Fr}^2)^{1/2}$. The substitutions carried out normalize the interval of unstable wave numbers for planar flow to unity. By substituting $h = h_0(\xi) + h'(\xi, t)$, $q = q_0(\xi) + q'(\xi, t)$ into the system (2) and then linearizing it, the following system of equations is obtained for investigating the stability of the steady-state solution $h_0(\xi)$, $q_0(\xi)$ (the primes are omitted on the perturbed quantities):

$$\frac{\partial q}{\partial t} - c\frac{\partial q}{\partial \xi} - 1{,}2\frac{q_0^2}{h_0^2}\frac{\partial h}{\partial \xi} + 2{,}4\frac{q_0}{h_0}\frac{\partial q}{\partial \xi} + \left(2{,}4\frac{\partial}{\partial \xi}\frac{q_0}{h_0} + \right. \qquad (3)$$
$$\left. + \frac{z}{h_0^2}\right) q - 3h_0\frac{\partial^3 h}{\partial \xi^3} - \left(1{,}2\frac{\partial}{\partial \xi}\frac{q_0^2}{h_0^2} + F + 2z\frac{q_0}{h_0^3} + 3\frac{\partial^3 h_0}{\partial \xi^3}\right) h = 0,\quad \frac{\partial h}{\partial t} - c\frac{\partial h}{\partial \xi} + \frac{\partial q}{\partial \xi} = 0.$$

Since the variable t does not appear explicitly here, then by representing the solution in the form $h = \exp(-\gamma t)h_1(\xi)$, $q = \exp(-\gamma t)q_1(\xi)$ a system of ordinary linear differential equations is obtained with periodic coefficients:

$$-\gamma q_1 - c\frac{\partial q_1}{\partial \xi} - 1{,}2\frac{q_0^2}{h_0^2}\frac{\partial h_1}{\partial \xi} + 2{,}4\frac{q_0}{h_0}\frac{\partial q_1}{\partial \xi} + \left(2{,}4\frac{\partial}{\partial \xi}\frac{q_0}{h_0} + \right.$$
$$\left. + \frac{z}{h_0^2}\right)q_1 - 3h_0\frac{\partial^3 h_1}{\partial \xi^3} - \left(1{,}2\frac{\partial}{\partial \xi}\frac{q_0^2}{h_0^2} + F + \right.$$
$$\left. + 2z\frac{q_0}{h_0^3} + 3\frac{\partial^3 h_0}{\partial \xi^3}\right)h_1 = 0, \quad -\gamma h_1 - c\frac{\partial h_1}{\partial \xi} + \frac{\partial q_1}{\partial \xi} = 0. \tag{4}$$

From Floquet's theorem it follows that the solution of the system (4) which is bounded for all values of ξ is of the form

$$h_1 = \varphi(\xi)\exp(i\alpha Q\xi), \quad q_1 = \psi(\xi)\exp(i\alpha Q\xi), \tag{5}$$

where φ, ψ are periodic functions of the same period as $h_0(\xi)$ and $q_0(\xi)$, and Q is a real parameter. By substituting (5) into (4) it is found that

$$-\gamma\psi + (A + i\alpha QB)\psi + B\frac{d\psi}{d\xi} - (P + i\alpha QD - 3i\alpha^3 Q^3 h_0)\varphi -$$
$$-(D - 9\alpha^2 Q^2 h_0)\frac{d\varphi}{d\xi} - 9i\alpha Q h_0\frac{d^2\varphi}{d\xi^2} - 3h_0\frac{d^3\varphi}{d\xi^3} = 0, \tag{6}$$
$$-\gamma\varphi + i\alpha Q\psi + \frac{d\psi}{d\xi} - i\alpha c Q\varphi - c\frac{d\varphi}{d\xi} = 0.$$

Thus the investigation of the stability of the steady-state wavy regime of $h_0(\xi)$, $q_0(\xi)$ can be reduced to the study for various values of Q of the spectrum of the eigenvalues of γ for which the system (6) has periodic solutions of the same period. The waves are stable if for any Q, all real $(\gamma) \geq 0$. It is clear from (5) that sufficient boundedness is given by the variation of Q in any unit interval, for instance [−0.5; 0.5]. By carrying out the operation of complex conjugation in the system (6), it is not difficult to show that $\gamma(Q) = \gamma(-Q)$, so that it is only necessary to consider the solution of (6) over the range $0 \leq Q \leq 0.5$.

The results obtained for Q = 0 give an answer to the problem of the stability with respect to a special but important type of perturbation, namely, perturbations of the same periodicity as the flow being investigated. In this case, one of the solutions of the system (6) can easily be found analytically: $\gamma = 0$, $\varphi = h_0'$, $\psi = q_0'$. This result is a direct consequence of the Andronov–Witt theorem on the existence of at least one zero Lyapunov exponent for the trajectory of the limiting cycle.

In order to find the remaining values of γ, the problem was solved numerically, as in the case when $Q \neq 0$. In order to do this, by carrying out a Fourier transformation on (6) an infinite system of linear homogeneous algebraic equations is obtained for φ_n, ψ_n. By assuming that φ_n, ψ_n are all equal to zero for $n \geq N$, we arrive at the final approximation:

$$\sum_{k=1}^{N}(V_{n-k+1+rN}\psi_k + W_{n-k+1+rN}\varphi_k) = \gamma\psi_n,$$
$$(i\alpha Q + i\alpha(n-1))\psi_n - (i\alpha Qc + i\alpha c(n-1))\varphi_n = \gamma\varphi_n, \tag{7}$$
$$n = 1, 2, \ldots, N, \quad r = \begin{cases} 0, & k \leqslant n, \\ 1, & k > n. \end{cases}$$

The number of harmonics N which must be taken into account depends on the solution h_0, q_0 being investigated. For waves of the first family with wave numbers α which are close

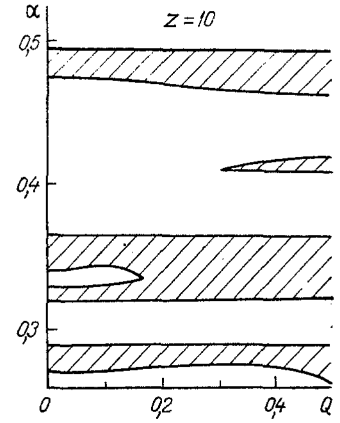

Fig. 4. Zones of stability with respect to all two-dimensional perturbations.

to the upper boundary of instability of planar flow ($\alpha \gtrsim 0.7$) it is sufficient to take N ∿ 8-16; as α decreases, N increases. For investigating the stability of waves of the second family (these waves are strongly nonlinear [4-7] it is customary to use 32 to 64 harmonics.

Let us discuss the results which have been obtained for the example of waves of the first type. The calculations show that the perturbations with the same periodicity are the least dangerous. Thus, while at Q = 0 the range of stability of the wave number α is quite wide (from the upper boundary of the linear instability of the plane-parallel flow, α = 1, to $\alpha \lesssim 0.5$) and exists for all values of z, for perturbations with $Q \neq 0$ the solution being studied is stable over a narrower range of wave numbers and only for quite large values of z (small Re). When $z \to \infty$, this range is transformed into the zone of stability found in [13] during the investigation of an equation which is valid for $Re \lesssim 1$.

Figure 2 gives the results for the first family of solutions with z = 10. Here the interval of stability with respect to perturbations with Q = 0 is $0.515 < \alpha < 1$, but the solutions which are stable to all two-dimensional perturbations have wave numbers $0.772 < \alpha < 0.801$.

As z decreases the zone of stable waves becomes narrower and is shifted towards smaller values of α. For z < 4 the zone of stability no longer reaches the axis Q = 0, i.e., at such values of z there are always values of Q for which the perturbations grow for any value of α.

Figure 2 also shows the zone in which the perturbations grow quite slowly. Here the growth increment is $\delta = -\mathrm{Real}(\gamma) < 10^{-2}$. For real liquids of the type of water–glycerol solutions the perturbations will only slightly disturb the wavy flow up to distances of ∿ 10-20 cm, and from an experimental point of view such regimes can be classified as steady-state travelling waves.

If is apparently reasonable assumption is made that imposed pulsations (giving rise to a wavy regime with a definite value of α) do not make it possible to realize perturbations with values of Q which differ strongly from zero, but necessarily (in view of the inevitable instability of the frequency of the driving pulsations) lead to perturbations with small values of Q, then it becomes possible to explain some of the experimental behavior. Thus, for instance, it is clear why for exactly the same wavelength the excited wave occurs at greater distances from the origin than the natural travelling wave: perturbations with values of Q which differ strongly from zero which are not forbidden for natural waves rapidly destroy them. For the same reason, excited waves are observed over a wider range of wave numbers. As a result of the closeness of the upper boundary of the zone of slowly growing perturbations to the limit of stability for all values $Q < Q_*$ (except for very small values) (it is assumed that in the generation of the excited waves it is possible to have small $Q > Q_*$), it is obvious that this widening of the zone takes place on the side of smaller wave numbers (see Fig. 3).

For this family of solutions it becomes practically impossible to observe the excited waves also when there is a shift of the boundary of stability relative to the perturbations of the same periodicity (Q = 0) as a result of the rapid growth of the increment. In particular, it is clear why in the experiments waves are not observed which are close to the sequence of negative soliton-depressions, though this is predicted by theory [3-7].

The results for the second family of solutions are qualitatively analogous, except that for them there are several stable zones (Fig. 4) and the characteristic increments of the most dangerous perturbations are smaller by almost an order of magnitude than in the case of the first family of solutions. These data give an answer to the problem of why preferentially highly linear wave elevations (almost to the extent of soliton waves) are observed in the experiments, particularly as Re increases.

NOTATION

ν, coefficient of kinematic viscosity; σ, surface tension coefficient; g, acceleration of free fall; ρ, liquid density; α, wave number; λ, wavelength; c, phase velocity of waves; $A_1 = h_{max} - h_{min}$; $We = \sigma\langle h\rangle/\rho\langle q\rangle^2$, Weber number; $Re = \langle q\rangle/\nu$, Reynolds number; $Fr = \langle q\rangle^2/g\langle h\rangle^2$, Froude number; Real (a), real part of the number a; $F_i = (\sigma/\rho)^3/g\nu^4$, film number; $A = 2.4\, d/d\xi(q_0/h_0) + z/h_0^2$; $B = 2.4\,(q_0/h_0) - c$; $p = 1.2\, d/d\xi(q_0^2/h_0^2) + F + 2zq_0/h_0^3 + 3\, d^3h_0/d\xi^3$; $D = 1.2\,(q_0^2/h_0^2)$; $V_{n-k+l+rN} = (A + i\alpha QB)_{n-k+l+rN} + i\alpha(k-1)B_{n-k+l+rN}$; $W_{n-k+l+rN} = (P + i\alpha QD - 3i\alpha^3Q^3h_0)_{n-k+l+rN} - i\alpha(k-1)(D - 9\alpha^2Q^2h_0)_{n-k+l+rN} + (9i\alpha^3Q(K-1)^2 + 3i\alpha^3(k-1)^3h_{0\,n-k+l+rN}$.

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STABILITY OF THE PROCESS OF EXTRUSION OF A VISCOELASTIC MATERIAL FROM A CONICAL CHANNEL

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UDC 532.5:532.135

The stability of motion of a viscoelastic compressible medium in a conical channel with a small outlet orifice is investigated during the initial stage of extrusion.

The motion of viscoelastic media through conical channels is a process encountered in plastics production – solid-phase extrusion, fiber-forming, etc. In studies of these processes and their stability the steady-state motion of incompressible media is usually considered. At the same time, polymeric materials cannot be regarded as perfectly incompressible (see [1, 2]) and, clearly, in the initial stage of extrusion through a spinneret, when the exit velocity has not yet reached its steady-state value, volume compression of

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